

# Информационные технологии в технической физике

#10 Оптимизация



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Оптимизационная задача

$$f(x) = \min_{x \in X}$$

Если у проблемы много решений, то  
**ОПТИМИЗАЦИЯ** — это поиск лучшего среди них

# # Оптимизационная задача





Пьер де Ферма  
Pierre de Fermat



Исаак Ньютон  
Isaac Newton

Условная

Безусловная

Локальная

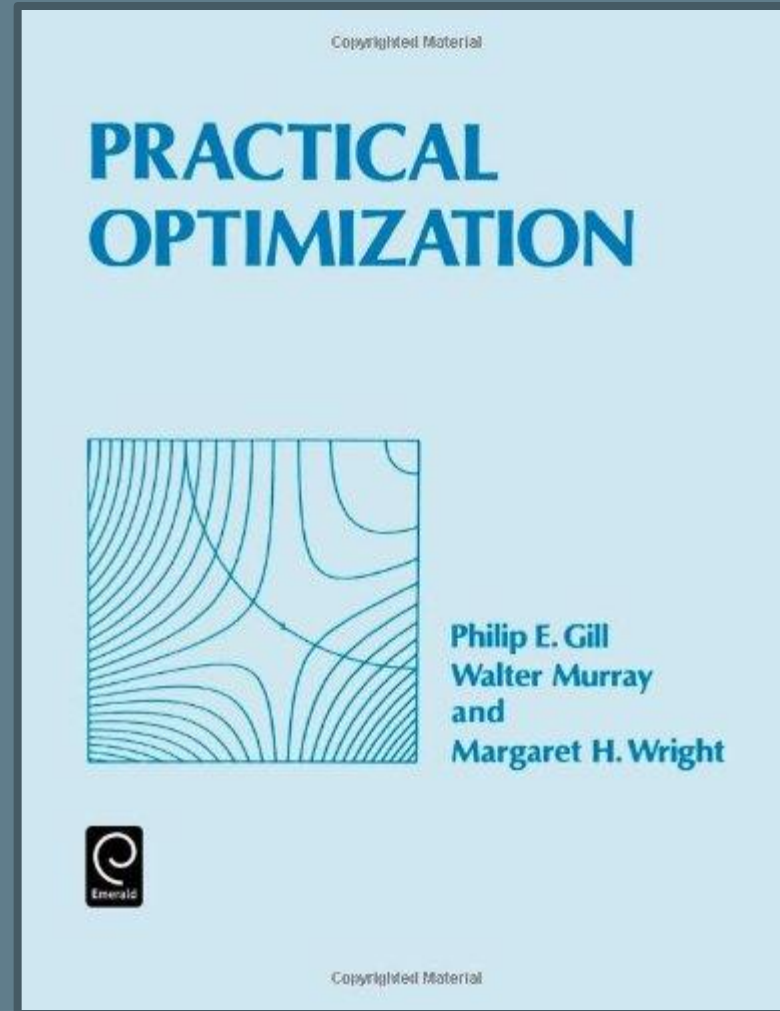
Глобальная

Условная

Безусловная

Локальная

Глобальная





### Optimization by Simulated Annealing

S. Kirkpatrick, C. D. Gelatt, Jr., M. P. Vecchi

In this article we briefly review the central constructs in combinatorial optimization and in statistical mechanics and then develop the similarities between the two fields. We show how the Metropolis algorithm for approximate numerical simulation of the behavior of a many-body system at a finite temperature provides a natural tool for bringing the techniques of statistical mechanics to bear on optimization.

We have applied this point of view to a number of problems arising in optimal design of computers. Applications to partitioning, component placement, and wiring of electronic systems are described in this article. In each context, we introduce the problem and discuss the improvements available from optimization.

Of classic optimization problems, the traveling salesman problem has received the most intensive study. To test the power of simulated annealing, we used the algorithm on traveling salesman problems with as many as several thousand cities. This work is described in a final section, followed by our conclusions.

#### Combinatorial Optimization

The subject of combinatorial optimization (1) consists of a set of problems that are central to the disciplines of computer science and engineering. Research in this area aims at developing efficient techniques for finding minimum or maximum values of a function of very many independent variables (2). This function, usually called the cost function or objective function, represents a quantitative mea-

sure of the "goodness" of some complex system. The cost function depends on the detailed configuration of the many parts of that system. We are most familiar with optimization problems occurring in the physical design of computers, so examples used below are drawn from

*Summary.* There is a deep and useful connection between statistical mechanics (the behavior of systems with many degrees of freedom in thermal equilibrium at a finite temperature) and multivariate or combinatorial optimization (finding the minimum of a given function depending on many parameters). A detailed analogy with annealing in solids provides a framework for optimization of the properties of very large and complex systems. This connection to statistical mechanics exposes new information and provides an unfamiliar perspective on traditional optimization problems and methods.

that context. The number of variables involved may range up into the tens of thousands.

The classic example, because it is so simply stated, of a combinatorial optimization problem is the traveling salesman problem. Given a list of  $N$  cities and a means of calculating the cost of traveling between any two cities, one must plan the salesman's route, which will pass through each city once and return finally to the starting point, minimizing the total cost. Problems with this flavor arise in all areas of scheduling and design. Two subsidiary problems are of general interest: predicting the expected cost of the salesman's optimal route, averaged over some class of typical arrangements of cities, and estimating or obtaining bounds for the computing effort necessary to determine that route.

All exact methods known for determining an optimal route require a computing effort that increases exponentially

with  $N$ , so that in practice exact solutions can be attempted only on problems involving a few hundred cities or less. The traveling salesman belongs to the large class of NP-complete (nondeterministic polynomial time complete) problems, which has received extensive study in the past 10 years (3). No method for exact solution with a computing effort bounded by a power of  $N$  has been found for any of these problems, but if such a solution were found, it could be mapped into a procedure for solving all members of the class. It is not known what features of the individual problems in the NP-complete class are the cause of their difficulty.

Since the NP-complete class of problems contains many situations of practical interest, heuristic methods have been developed with computational require-

ments proportional to small powers of  $N$ . Heuristics are rather problem-specific; there is no guarantee that a heuristic procedure for finding near-optimal solutions for one NP-complete problem will be effective for another.

There are two basic strategies for heuristics: "divide-and-conquer" and iterative improvement. In the first, one divides the problem into subproblems of manageable size, then solves the subproblems. The solutions to the subproblems must then be patched back together. For this method to produce very good solutions, the subproblems must be naturally disjoint, and the division made must be an appropriate one, so that errors made in patching do not offset the gains

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Метод отжига – это оптимизационный метод, основанный на случайном поиске, по аналогии с процессом кристаллизации вещества при охлаждении

- Выбирается случайная точка, устанавливается значение энергии
- ...
- Шаг итерации:
  - Вычисляется значение энергии в данной **точке**
  - Выбирается новое минимальное значение энергии между текущим минимумом и данной **точкой**
  - Генерируется новая **точка**, вычисляется энергия в данной **точке**
  - Вычисляется вероятность перехода в новое состояние при заданной температуре
  - Переход или генерация ещё одной новой **точки**
- ...
- Достигли минимальной температуры

- Больцмановский отжиг
- Отжиг Коши
- Отжиг Инберга



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