Nernst heat theorem for the Casimir-Polder interaction between a magnetizable atom and ferromagnetic dielectric plate

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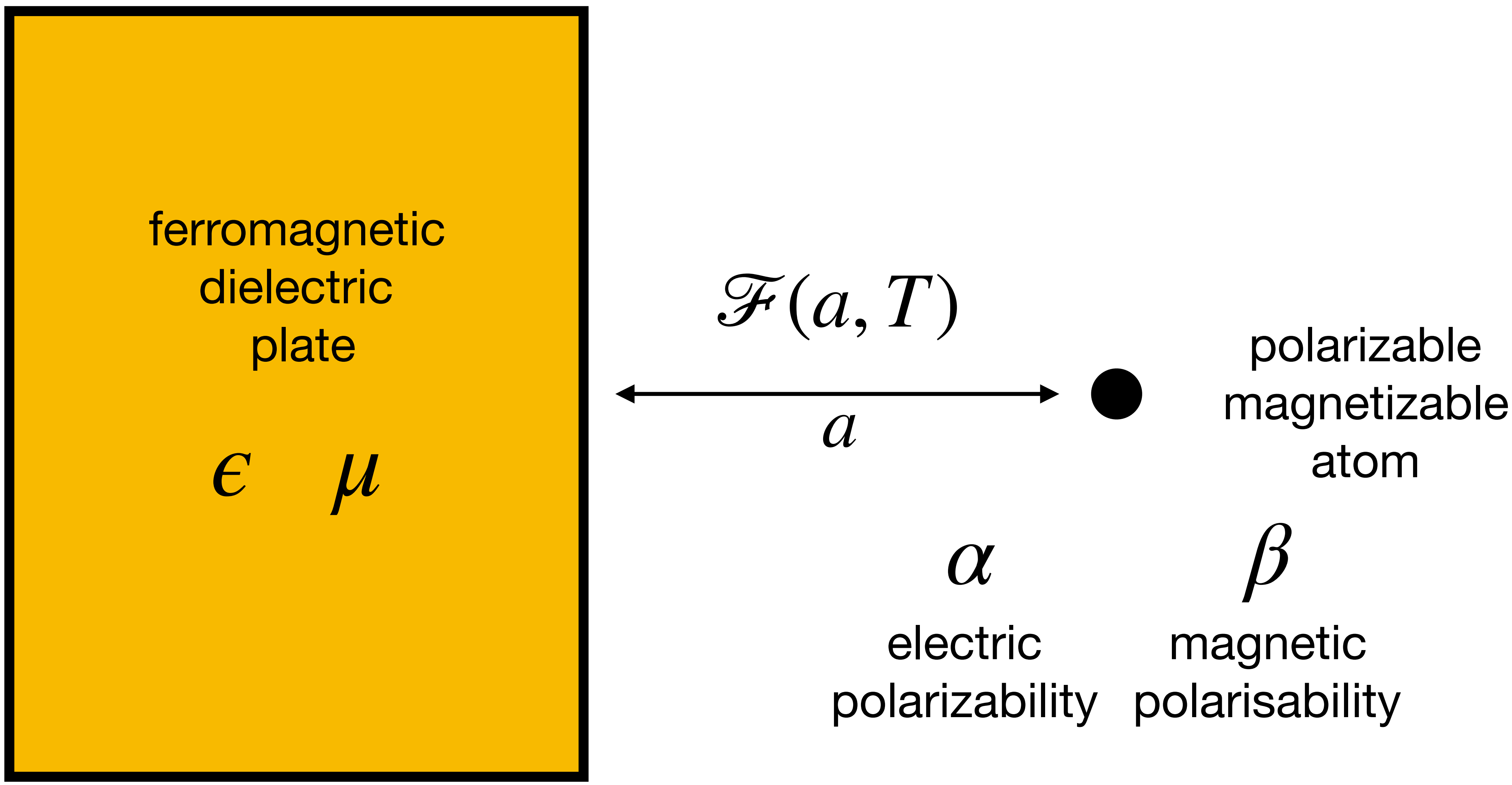
1. Introduction

The Casimir-Polder interaction



Motivation

Puzzle

There is a disagreement of the measurement data with the Lifshitz theory using the Drude model (metals) and dielectric model with taken into account conductivity at a constant current (dielectrics). The respective Casimir free energy and entropy violate the Nernst heat theorem.

Problem

Does the Casimir-Polder entropy satisfy the Nernst heat theorem for a system of polarizable and magnetizable atom interacting with a plate made of ferromagnetic dielectric?

2. Lifshitz theory for Casimir-Polder interaction

Lifshitz formula

$$\mathcal{F}(a,T) = \frac{k_B T}{8a^3} {\sum_{l=0}^{\infty}}' \left\{ \Phi^{(\alpha)}(\zeta_l) + \Phi^{(\beta)}(\zeta_l) \right\},$$

$$\Phi^{(\alpha)}(\zeta_l) = -\,\alpha_l \int_{\zeta_l}^{\infty} dy\; e^{-y} \left[2y^2 r_{\mathrm{TM},l} - \zeta_l^2 (r_{\mathrm{TM},l} + r_{\mathrm{TE},l}) \right],$$

$$\Phi^{(\beta)}(\zeta_l) = -\,\beta_l \int_{\zeta_l}^{\infty} dy\; e^{-y} \left[2y^2 r_{\mathrm{TE},l} - \zeta_l^2 (r_{\mathrm{TM},l} + r_{\mathrm{TE},l}) \right].$$

Matsubara frequencies

$$\zeta_l = \xi_l/\omega_c, \quad \omega_c = c/(2a)\,.$$

k_B is the Boltzmann constant

The reflection coefficients of the plate

$$r_{\mathrm{TM},l} = r_{\mathrm{TM}}(i\zeta_l, y) = \frac{\epsilon_l y - \sqrt{y^2 + \zeta_l^2 \gamma_l}}{\epsilon_l y + \sqrt{y^2 + \zeta_l^2 \gamma_l}},$$

$$r_{\mathrm{TE},l} = r_{\mathrm{TE}}(i\zeta_l, y) = \frac{\mu_l y - \sqrt{y^2 + \zeta_l^2 \gamma_l}}{\mu_l y + \sqrt{y^2 + \zeta_l^2 \gamma_l}}.$$

$$\gamma_l = \epsilon_l \mu_l - 1, \quad \epsilon_l = \epsilon(i\omega_c \zeta_l), \quad \mu_l = \mu(i\omega_c \zeta_l).$$

3. The Casimir-Polder free energy: Static case

Model

$$\epsilon_0, \quad \mu_0, \quad \alpha_0, \quad \beta_0 \,.$$

Abel-Plana formula

$$\Delta\mathcal{F}(a,T) = \frac{ik_BT}{8a^3}\int_0^\infty \mathrm{d}t\, \frac{\Phi(i\tau t) - \Phi(-i\tau t)}{\mathrm{e}^{2\pi t} - 1}, \qquad \Phi(x) = -\,\alpha_0 I(x,\epsilon_0) - \beta_0 I(x,\mu_0),$$

$$I(x,\eta) = x^3 \int_1^\infty \mathrm{d}z\, \mathrm{e}^{-xz}\left[2z^2 r(\eta,z) - r(\epsilon_0,z) - r(\mu_0,z)\right],$$

$$z = y/x, \qquad\qquad r(\eta,z) = \frac{\eta z - \sqrt{z^2+\gamma_0}}{\eta z + \sqrt{z^2+\gamma_0}}\,.$$

$$I(x,\eta) = x^3 \left[I_1(x,\eta) + I_2(x,\eta) + I_3(x,\eta)\right],$$

$$I_1(x,\eta) = 2\int_1^\infty \mathrm{d}z\ \mathrm{e}^{-xz} \left\{ z^2 \left[r(\eta,z) - \frac{\eta-1}{\eta+1}\right] + \frac{\eta\gamma_0}{(\eta+1)^2} \right\},$$

$$I_2(x) = \int_1^\infty \mathrm{d}z\ \mathrm{e}^{-xz} \left[-r(\epsilon_0,z) + \frac{\epsilon_0-1}{\epsilon_0+1} - r(\mu_0,z) + \frac{\mu_0-1}{\mu_0+1}\right],$$

$$I_3(x,\eta) = \int_1^\infty \mathrm{d}z\ \mathrm{e}^{-xz} \left[2z^2\frac{\eta-1}{\eta+1} - \frac{2\eta\gamma_0}{(\eta+1)^2} - \frac{\epsilon_0-1}{\epsilon_0+1} - \frac{\mu_0-1}{\mu_0+1}\right].$$

The temperature-dependent contribution to the Casimir-Polder free energy

$$\Delta\mathcal{F}(a,T) = -\frac{\pi^3(k_BT)^4}{15(\hbar c)^3}\Big[\alpha_0 B_\alpha(\epsilon_0,\mu_0) + \beta_0 B_\beta(\epsilon_0,\mu_0)\Big],$$

$$B_\alpha \xrightarrow[\epsilon_0\leftrightarrow\mu_0]{} B_\beta\,.$$

$$
\begin{aligned}
B_\alpha(\epsilon_0,\mu_0) = {} & \frac{\mu_0^2+1}{\mu_0^2-1} + \frac{\epsilon_0^4 - 12\epsilon_0^2\gamma_0 - 1}{3(\epsilon_0^2-1)^2} \\
& -2\sqrt{\epsilon_0\mu_0}\left[\frac{\epsilon_0}{3\left(\epsilon_0^2-1\right)} + \frac{\mu_0}{\mu_0^2-1} - \frac{2}{3}\frac{\epsilon_0(\epsilon_0^2+2)\gamma_0}{(\epsilon^2-1)^2}\right] \\
& -\frac{2\mu_0\sqrt{\gamma_0}}{(\mu_0^2-1)^{3/2}}\ln\frac{\sqrt{\gamma_0}+\sqrt{\mu_0^2-1}}{\mu_0\sqrt{\gamma_0}+\sqrt{\epsilon_0\mu_0}\sqrt{\mu_0^2-1}} \\
& +\frac{2\epsilon_0^2\sqrt{\gamma_0}(2\epsilon_0\mu_0-1-\epsilon_0^2)}{(\epsilon_0^2-1)^{5/2}}\ln\frac{\sqrt{\gamma_0}+\sqrt{\epsilon_0^2-1}}{\epsilon_0\sqrt{\gamma_0}+\sqrt{\epsilon_0\mu_0}\sqrt{\epsilon_0^2-1}}\,.
\end{aligned}
$$

4. The Casimir-Polder free energy: Influence of dc conductivity

Model

$$\widetilde{\epsilon}_l = \epsilon_l + \frac{4\pi\widetilde{\sigma}_0(T)}{\zeta_l}, \quad \mu_0, \quad \alpha_0, \quad \beta_0 \,.$$

The Reflection coefficients of the plate

$$r_{\mathrm{TM},0} = \frac{\epsilon_0 - 1}{\epsilon_0 + 1}, \quad \widetilde{r}_{\mathrm{TM},0} = 1, \quad r_{\mathrm{TE},0} = \widetilde{r}_{\mathrm{TE},0} \,.$$

B. Geyer, G. L. Klimchitskaya and V. M. Mostepanenko, Phys. Rev. D 72, 085009 (2005)

The Casimir-Polder free energy

$$\widetilde{\mathcal{F}}(a,T) = \mathcal{F}(a,T) - \frac{k_B T}{4a^3}\left(1 - \frac{\epsilon_0 - 1}{\epsilon_0 + 1}\right)\alpha_0 \,.$$

5. Nernst heat theorem

The Casimir-Polder entropy without dc conductivity

$$S(T) = \frac{4\pi^3}{15(\hbar c)^3} k_B (k_B T)^3 \left[\alpha_0 B_\alpha(\epsilon_0, \mu_0) + \beta_0 B_\beta(\epsilon_0, \mu_0) \right].$$

$$\boxed{S(0) = 0}$$

The Nernst heat theorem is satisfied

The Casimir-Polder entropy with dc conductivity

$$\widetilde{S}(a,T) = S(T) + \frac{k_B \alpha_0}{2a^3} \frac{1}{\epsilon_0 + 1}\,.$$

$$\boxed{\widetilde{S}(a,0) = \frac{k_B \alpha_0}{2a^3} \frac{1}{\epsilon_0 + 1} > 0}$$

The Nernst heat theorem is violated
(entropy at T=0 depends on the parameters on a system)

6. Conclusions

1. We have found an asymptotic behaviour of the Casimir-Polder free energy and entropy for the polarizable and magnetizable atom interacting with a ferromagnetic dielectric plate at arbitrarily low temperature using the Lifshitz theory.

2. It is shown that the Nernst heat theory is satisfied if the dc conductivity of plate material is disregarded in calculations and is violated otherwise.

3. This raises the same problem of thermodynamic consistency as was discussed earlier for other Casimir and Casimir-Polder configurations.

Thank you for your attention!